Exercise 97

The number of bacteria in a refrigerated food product is given by $N(T) = 23T^2 - 56T + 1$, $3 < T < 33$, where T is the temperature of the food. When the food is removed from the refrigerator, the temperature is given by $T(t) = 5t + 1.5$, where t is the time in hours.

(a) Find the composite function $N(T(t))$.

(b) Find the time (round to two decimal places) when the bacteria count reaches 6752.

Solution

Plug the formula for $T(t)$ into the formula for $N(T)$ and simplify.

$$\begin{aligned} N(T(t)) &= 23[T(t)]^2 - 56[T(t)] + 1 \\ &= 23(5t + 1.5)^2 - 56(5t + 1.5) + 1 \\ &= 23(25t^2 + 15t + 2.25) - 280t - 84 + 1 \\ &= 575t^2 + 345t + 51.75 - 280t - 83 \\ &= 575t^2 + 65t - 31.25 \end{aligned}$$

Therefore,

$$\boxed{N(T(t)) = 575t^2 + 65t - 31.25.}$$

To get the time when the bacteria count reaches 6752, set $N = 6752$ and solve the equation for t.

$$6752 = 575t^2 + 65t - 31.25$$

Subtract 6752 from both sides.

$$575t^2 + 65t - 6783.25 = 0 \tag{1}$$

Use the quadratic formula.

$$t = \frac{-65 \pm \sqrt{65^2 - 4(575)(-6783.25)}}{2(575)} = \frac{-65 \pm \sqrt{15605700}}{1150}$$

The two times that satisfy equation (1) are

$$t = \frac{-65 + \sqrt{15605700}}{1150} \approx 3.38$$

$$t = \frac{-65 - \sqrt{15605700}}{1150} \approx -3.49.$$

The second time is negative and can be discarded. Therefore, it takes about 3.38 hours for the bacteria count to reach 6752.